

## Power flow analysis with easy modelling of interline power flow controller

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### ARTICLE INFO

#### Article history:

Received 21 January 2013

Received in revised form

18 November 2013

Accepted 21 November 2013

Available online 20 December 2013

#### Keywords:

FACTS

IPFC

Newton Raphson power flow

Revised current injection load flow

### ABSTRACT

This paper proposes an easy modelling of interline power flow controller (IPFC) into Revised Newton Raphson current injection load flow method. In this model, the IPFC is represented as series impedances with shunt injected currents at its terminal buses. The target of control for active and reactive power flow can be achieved by calculating these currents as a function of the desired power flow and the buses voltage at the terminals of IPFC. In case of controlling the active power flow only, these currents are calculated with the same method. But the reactive power flow is released and calculated according to the system. The injected currents are updated and added into the original current mismatches vector of load flow algorithm. By using this model, the symmetry of the admittance and Jacobian matrices can still be kept and incorporating of IPFC becomes easy without changing the basic load flow computational program. Consequently, the complexities of the computer program codes are reduced. Numerical results based on the literature 5-bus, IEEE 57-bus and IEEE 118-bus systems are used to demonstrate the effectiveness and performance of the proposed IPFC model.

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## 1. Introduction

Flexible AC transmission system controllers are playing a leading role in efficiently controlling the line power flow and improving voltage profiles of the power system network. These FACTS controllers can be used to increase the reliability and efficiency of transmission and distribution systems [1–7].

In general, there are two generations of these developed control devices; the first generation is based on the conventional thyristor switched capacitors and reactors, and tap changing transformers, while, the second generation uses the GTO thyristor switched converters as voltage source converters (VSCs).

The first generation has resulted in the SVC, TCSC and the TCPS. The second generation has produced the STATCOM, the SSSC, the UPFC and the IPFC. The two groups of FACTS controllers have distinctly different operating and performance characteristics [4].

The IPFC is considered a new generation of FACTS controllers. The combinations of two or more series compensators are coupled via a common dc link to give the structure of IPFC.

The main advantage of IPFC is the ability to control both real and reactive power flow at a multi-line substation. Hence, the power can be transferred from the over loaded line to the under loaded line [7].

In general, the modelling of FACTS in power flow calculations with improving the reusability of computer program codes and avoiding the Jacobian modifications become important and challenging research problem. In contrast to the modelling of the UPFC and SSSC, research work on the modelling of IPFC for power system analysis is limited.

However, the following modifications are required in load flow analysis in order to incorporate FACTS controllers: firstly, the incorporation of FACTS into a transmission line requires adding auxiliary buses in the system. Secondly, the FACTS impedances have to be included into the admittance. Thirdly, the powers contributed by FACTS have to be included into power flow mismatch equations. Finally, the system Jacobian matrix contains entirely new Jacobian sub-blocks exclusively related to the FACTS controllers [4].

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## Nomenclature

p.u	per unit
NR	Newton–Raphson method
PV	voltage controlled buses
PQ	load buses
FACTS	flexible AC transmission systems
IPFC	interline power flow controller
STATCOM	static synchronous compensator
UPFC	unified power-flow controller
SSSC	static series synchronous compensator
TCSC	thyristor controlled series capacitor
SVC	static var compensator
GTO	gate turn-off
VSCs	voltage source converters
TCPS	thyristor-controlled phase shifter
<i>N</i>	total number of buses
<i>P, Q</i>	active and reactive complex powers
$\Delta I_{rf} + j\Delta I_{mf}$	complex current mismatch at bus <i>f</i>
$\Delta V_{rf} + j\Delta V_{mf}$	complex voltage mismatch at bus <i>f</i>
<i>r, m</i>	subscripts refer to real and imaginary parts
<i>i, j, k</i>	subscripts refer to nodes
<i>sp</i>	superscript refers to specified values
$Z_{se1}$	the series transformer impedance of main converter
$Z_{se2}$	the series transformer impedance of slave converter
$V_i, V_j, V_k$	the complex voltages at sending and auxiliary buses
$V_{se1}$	the complex series injected voltage source for main converter
$V_{se2}$	the complex series injected voltage source for slave converter
$G_{km} + jB_{km}$	the element of nodal admittance matrix between buses ( <i>k, m</i> )
$P_{G(f)} + jQ_{G(f)}$	generated complex power at bus <i>f</i>
$P_{L(f)} + jQ_{L(f)}$	complex power consumed by load at bus <i>f</i>
$\partial$	refers to partial derivatives

Due to the above requirements, many excellent research works have been carried to incorporate some famous FACTS with minimum modifications in original load flow algorithm [7–23]. However, these developed techniques can be applied for modelling IPFC controller in load flow analysis.

Ref. [8] has presented simple modelling of FACTS based on decoupled approach. In this technique, the sending and receiving buses of FACTS are separated. Then, active and reactive loads are injected at the terminal buses with desired line flow. To control the voltage at particular bus, this bus has to be converted into PV-type with required voltage value. Then, after load flow convergence is achieved, an additional set of non-linear equations related to various FACTS parameters has to be solved. However, this technique faced some shortcomings such as; the FACTS parameters are computed after the load flow converge, so, the method has not the ability to known whether these parameters are within Limits or not, the method discussed only the situations when the FACTS device is used to control the active power flow, reactive power flow and voltage simultaneously, also, the problem of selecting suitable initial value of FACTS parameters still exists. Finally, what is the solution when the FACTS device is the only link between two sub-networks.

Ref. [12] has presented the FACTS control parameters as independent variables and their values are calculated during the iterative process of load flow program. This technique increases the size of the Jacobian matrix in order to accommodate the additional independent state variables of FACTS devices. However, the convergence manner is very sensitive to the initial value of the FACTS parameters [13]. Also the Jacobian matrix should be changed related to contribution of FACTS device.

Modelling of FACTS controllers based on indirect approach to reduce the complexity of programming codes have been presented in Refs. [16–18]. In this technique, the FACTS device is presented by an augmented equivalent network. Then, without any FACTS, the standard NR load flow can be carried out to calculate the steady state operating point of the original system containing FACTS. In this technique, the size of the Jacobian matrix has to be increased in order to accommodate the additional state variables of FACTS.

Modelling of FACTS controllers based on power injection approach has been presented in Refs. [14,15]. In these models, the Jacobian matrix can keep the block-diagonal properties; the FACTS state variables are adjusted simultaneously with the network state variables in order to achieve the specified control targets. In these two above methods, the elements of Jacobian matrix contain entirely new Jacobian sub-blocks exclusively related to the FACTS controllers have to be updated during the iterative process.

Ref. [19] presents an elegant approach based on power injection formulation to model UPFC in load flow algorithm to avoid the modification in Jacobian matrix.

With respect to load flow problem, NR method is considered as the state of the art load flow technique and widely used in the industry. The main disadvantage of this technique is the necessity for factorizing and updating the Jacobian matrix during the iterative process. However, the FD load flow method was proposed to speed up the NR load flow method and decreases the required minimum memory storage [24]. The main disadvantage of this method is affecting of convergence rate with high R/X ratios [25].

NR current injection mismatches load flow has been proposed to solve the problem of updating the Jacobian elements in conventional NR load flow [26–30]. In this approach, the Jacobian matrix can be obtained faster than the conventional Jacobian matrix especially in case of PQ buses. Where, in this case the elements of the Jacobian matrix are constant and equal to the admittance matrix.

In case of PV buses, the formulation has a low convergence rate. Hence, a revised NR current injection mismatches load flow has been presented to improve the convergence characteristics of in case of PV-type [30]. In this formulation, the PV bus is treated as PQ bus while taking the reactive power as an additional state variable.

Based on the review above, this paper describes a developed model for IPFC controller in Revised Newton Raphson current injection load flow method. The model is based on current injection approach. In this model, the original structure and symmetry of the admittance matrix can still be kept, the Jacobian matrix can keep without changing and sparsity technique can be easily applied.

The IPFC is represented as series impedances with injected currents at its terminal buses. The target of control for active and reactive power flows in the master line can be achieved by updating these shunt injected currents as a function of the desired line flow and the buses voltage of the terminals of IPFC. Also the target of control for the active line flow in the slave line can be achieved by calculating the shunt injected currents with the same technique but with line reactive power flow.

These injected currents are updated and added to the original mismatches of Revised NR current injection load flow method. In this model, an incorporating of IPFC in load flow becomes easy without changing in the basic computational algorithm. Consequently, the complexities of the computer program codes are reduced.

## 2. Newton Raphson current injections load flow method

A new development on Newton Raphson load flow based on current injections formulation was presented in Ref. [26]. In this development, the Jacobian matrix can be obtained faster than the conventional Jacobian of NR which based on power mismatches. The most elements of this Jacobian matrix are constant. Where, the off-diagonal elements plus a few diagonal ones equal to corresponding elements of the nodal admittance matrix. In case of PV buses, the calculation of the related elements of Jacobian matrix needs additional effort. However, this load flow method does not require at all the use of transcendental functions during the iterative process. The PV, PQ buses are represented by two equations comprising the real and imaginary components of current injection mismatches expressed in terms of the voltage rectangular coordinates.

In the first version of this formulation, a new dependent variable ( $\Delta Q$ ) is introduced for each PV bus together with an additional equation imposing the constraint of zero deviations in the bus voltage.

Revised version of this load flow method has presented in Ref. [30]. This formulation tried to solve the drawback of the original representation of PV buses in the first version which was based on the assumption that the voltage mismatches of PV buses equal to zero from the first iteration [26]. This condition is only true after load flow convergence. In this version, an additional equation has been presented to represent the voltage mismatch for each PV bus.

In this paper, the revised version of this load flow method is used. The algorithm of Revised Newton Raphson load flow based on current injection mismatches approach is given by Eq. (1):

$$\begin{bmatrix} \Delta I_{m1} \\ \Delta I_{r1} \\ \Delta I_{m2} \\ \Delta I_{r2} \\ \vdots \\ \Delta I_{mf} \\ \Delta I_{rf} \\ \Delta V_f^2 \\ \vdots \\ \Delta I_{mn} \\ \Delta I_m \end{bmatrix} = \begin{bmatrix} B'_{11} & G'_{11} & B_{12} & G_{12} & \cdots & B_{1f} & G_{1f} & 0 & \cdots & B_{1n} & G_{1n} \\ G''_{11} & B''_{11} & G_{12} & -B_{12} & \cdots & G_{1f} & -B_{1f} & 0 & \cdots & G_{12} & -B_{1n} \\ B_{21} & G_{21} & B'_{22} & G'_{22} & \cdots & B_{2f} & G_{2f} & 0 & \cdots & B_{2n} & G_{2n} \\ G_{21} & -B_{21} & G''_{22} & B''_{22} & \cdots & G_{2f} & -B_{2f} & 0 & \cdots & G_{2n} & -B_{2n} \\ \vdots & 0 & \vdots & \vdots & \vdots \\ B_{f1} & G_{f1} & B_{f2} & G_{f2} & \cdots & B'_{ff} & G'_{ff} & \frac{V_{rf}}{V_f^2} & \cdots & B_{fn} & G_{fn} \\ G_{f1} & -B_{f1} & G_{f2} & -B_{f2} & \cdots & G''_{ff} & B''_{ff} & \frac{-V_{mf}}{V_f^2} & \cdots & G_{fn} & -B_{fn} \\ 0 & 0 & 0 & 0 & \cdots & 2V_{rf} & 2V_{mf} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots \\ B_{n1} & G_{n1} & B_{n2} & G_{n2} & \cdots & B_{nf} & G_{nf} & 0 & \cdots & B'_{nn} & G'_{nn} \\ G_{n1} & -B_{n1} & G_{n2} & -B_{n2} & & G_{nf} & -B_{nf} & 0 & & G''_{nn} & B''_{nn} \end{bmatrix} \begin{bmatrix} \Delta V_{r1} \\ \Delta V_{m1} \\ \Delta V_{r2} \\ \Delta V_{m2} \\ \vdots \\ \Delta V_{rf} \\ \Delta V_{mf} \\ \Delta Q_f \\ \vdots \\ \Delta V_{rn} \\ \Delta V_{mn} \end{bmatrix} \quad (1)$$

where,

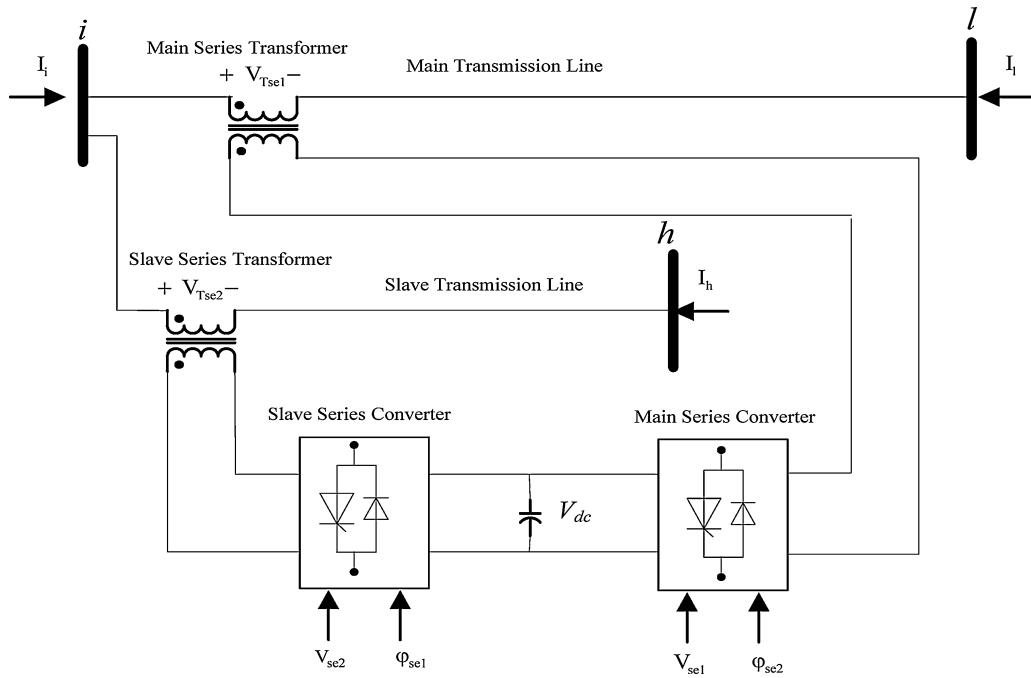
$$B'_{ff} = B_{ff} - a_f \quad (2)$$

$$G'_{ff} = G_{ff} - b_f \quad (3)$$

$$B''_{ff} = -B_{ff} - d_f \quad (4)$$

$$G''_{ff} = G_{ff} - c_f \quad (5)$$

The parameters ( $a_f, b_f, c_f, d_f$ ) are given in Ref. [26].



**Fig. 1.** Schematic diagram of two converters IPFC controller.

### 3. Developed current injection model of IPFC controller

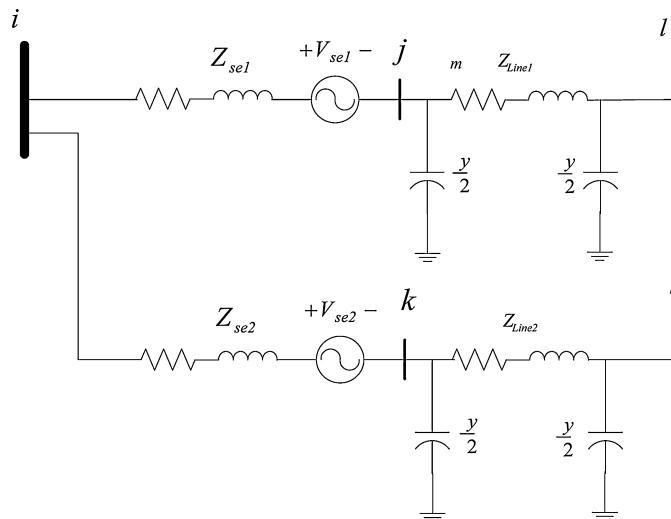
The combination of two or more static synchronous series compensators (SSSCs) coupled via a common dc link facilitates the bi-directional flow of active power between the ac terminals of the SSSCs, provides an independent reactive compensation for the adjustment of active power flow in each line and maintains the required distribution of reactive power flow among the lines.

The developed model is applied on the simplest IPFC that consists of two back-to-back dc-ac converters [1,3,7]. However, this model is applicable to an IPFC with any number of series converters. Fig. 1 shows the basic operation principles of IPFC controller.

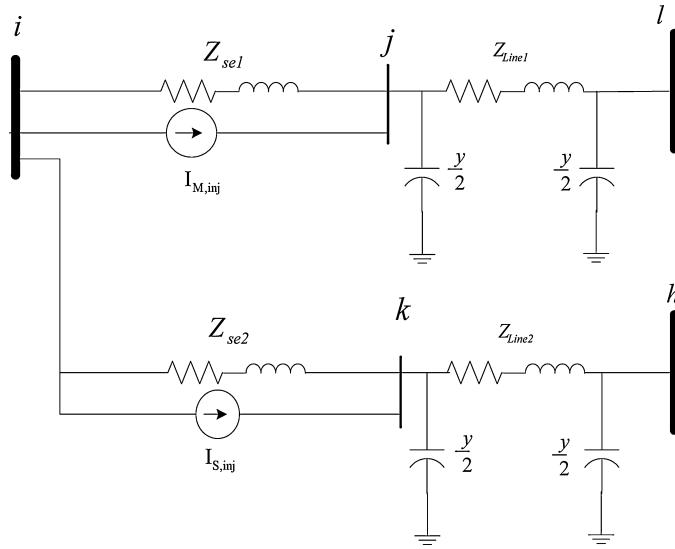
The equivalent circuit of the IPFC with two converters is represented by two controllable series-injected voltage sources, as shown in Fig. 2. Real power can be exchanged between two or among three or more series converters via the common dc link, and the sum of this exchanged real power should be zero.

The IPFC can be used to control the active and reactive power flows of the master line (M) and active or reactive power flow of the slave line (S). In general, the connected IPFC lines can be taken as master or slave according to the actual demand. In case of slave line, the active power is controlled while the reactive power is released.

The steady-state IPFC mathematical current injection model is developed by converting the voltage sources into current sources parallel with the series impedance of IPFC model as shown in Fig. 3. Also the parallel current source can be transformed to shunt injection current at the terminals of IPFC as shown in Fig. 4. The values of these current sources are dependent on the desired power flow and the voltage buses of the ends of IPFC. The injected shunt currents can be calculated as follows:



**Fig. 2.** Equivalent circuit of two converters IPFC controller.



**Fig. 3.** Current source representation of IPFC controller.

### 3.1. Master line in IPFC model

**Fig. 4** shows the current injection equivalent circuit of IPFC model. The shunt injected current at bus  $j$  can be calculated using Kirchhoff's current law as follows:

$$I_{M,inj} = I_{sp}^M - I_{ij} \quad (6)$$

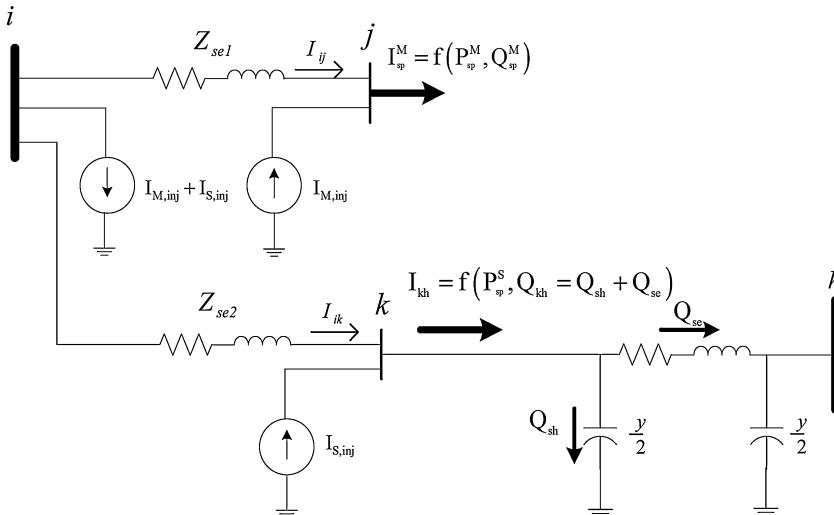
$$I_{M,inj} = \left( \frac{S_{sp}^M}{V_j} \right)^* - \left( \frac{V_i - V_j}{Z_{se1}} \right) \quad (7)$$

Where, ( $S_{sp}^M = P_{sp}^M + jQ_{sp}^M$ ,  $V_i = V_{ri} + jV_{mi}$ ,  $V_j = V_{rj} + jV_{mj}$ ,  $Z_{se1} = R_{se1} + jX_{se1}$ )  
The injected current can be rewritten as given in Eq. (8).

$$I_{M,inj} = \frac{(P_{sp}^M V_{rj} + Q_{sp}^M V_{mj}) + j(P_{sp}^M V_{mj} - Q_{sp}^M V_{rj})}{V_j^2} - \frac{[R_{se1}(V_{ri} - V_{rj}) + X_{se1}(V_{mi} - V_{mj})]}{Z_{se1}^2} - j \frac{[R_{se1}(V_{mi} - V_{mj}) - X_{se1}(V_{ri} - V_{rj})]}{Z_{se1}^2} \quad (8)$$

Then, the real and imaginary components of the injected current can be calculated as:

$$I_{M,inj}^{Re} = \frac{(P_{sp}^M V_{rj} + Q_{sp}^M V_{mj})}{V_j^2} - \frac{[R_{se1}(V_{ri} - V_{rj}) + X_{se1}(V_{mi} - V_{mj})]}{Z_{se1}^2} \quad (9)$$



**Fig. 4.** Current injection model of IPFC controller.

$$I_{M,inj}^{Im} = \frac{(P_{sp}^M V_{mj} - Q_{sp}^M V_{rj})}{V_j^2} - \frac{[R_{se1}(V_{mi} - V_{mj}) - X_{se1}(V_{ri} - V_{rj})]}{Z_{se1}^2} \quad (10)$$

This current is injected at the first auxiliary bus ( $j$ ) and also at the sending bus ( $i$ ) but with opposite sign

### 3.2. Slave line in IPFC model

By using the same mathematical analysis in case of master line, the shunt injected current at bus ( $k$ ) can be calculated using Eq. (11).

$$I_{S,inj} = \left( \frac{S_{sp}^S}{V_k} \right)^* - \left( \frac{V_i - V_k}{Z_{se2}} \right) \quad (11)$$

where, ( $S_{sp}^S = P_{sp}^S + jQ_{kh}$ ,  $V_i = V_{ri} + jV_{mi}$ ,  $V_k = V_{rk} + jV_{mk}$ ,  $Z_{se2} = R_{se2} + jX_{se2}$ )

The reactive power flow ( $Q_{kh}$ ) is considered the main difference between the main line and slave line. Where, in the main line is specified but in slave line is uncontrolled and can be calculated as follows:

$$Q_{kh} = Im(V_k I_{kh}^*) \quad (12)$$

where,  $I_{kh}$  can be calculated using Kirchhoff's current law from Fig. 4.

$$I_{kh} = j \left( \frac{B}{2} \right) V_k + Y_{kh}(V_k - V_h) \quad (13)$$

Then,

$$Q_{kh} = Im \left[ V_k \left( j \left( \frac{B}{2} \right) V_k + Y_{kh}(V_k - V_h) \right)^* \right] \quad (14)$$

$$Q_{kh} = V_{mk} \left[ G_{kh}(V_{rk} - V_{rh}) - B_{kh}(V_{mk} - V_{mh}) - V_{mk} \left( \frac{B}{2} \right) \right] - V_{rk} \left[ G_{kh}(V_{mk} - V_{mh}) + B_{kh}(V_{rk} - V_{rh}) + V_{rk} \left( \frac{B}{2} \right) \right] \quad (15)$$

The shunt injected current at bus ( $k$ ) can be calculated as a function of the specified power flow, the terminals' buses voltage of IPFC and the system reactive power flow between buses ( $k$ ) and ( $h$ ) as presented in Eq. (16).

$$I_{S,inj} = \frac{(P_{sp}^S V_{rk} + Q_{kh}^S V_{mk}) + j(P_{sp}^S V_{mk} - Q_{kh}^S V_{rk})}{V_j^2} - \frac{[R_{se2}(V_{ri} - V_{rk}) + X_{se2}(V_{mi} - V_{mk})]}{Z_{se2}^2} - j \frac{[R_{se2}(V_{mi} - V_{mk}) - X_{se2}(V_{ri} - V_{rk})]}{Z_{se2}^2} \quad (16)$$

Then, the real and imaginary components of shunt injected current at bus ( $k$ ) can be calculated as follows:

$$I_{S,inj}^{Re} = \frac{(P_{sp}^S V_{rk} + Q_{kh}^S V_{mk})}{V_k^2} - \frac{[R_{se2}(V_{ri} - V_{rk}) + X_{se2}(V_{mi} - V_{mk})]}{Z_{se2}^2} \quad (17)$$

$$I_{S,inj}^{Im} = \frac{(P_{sp}^S V_{mk} - Q_{kh}^S V_{rk})}{V_k^2} - \frac{[R_{se2}(V_{mi} - V_{mk}) - X_{se2}(V_{ri} - V_{rk})]}{Z_{se2}^2} \quad (18)$$

This current is injected at the second auxiliary bus ( $k$ ) and also at the sending bus ( $i$ ) but with opposite sign.

**Table 1**

The calculation of series voltage parameters of IPFC model.

Master converter	Slaver converter
$V_{se}^M = I_{M,inj}Z_{se1}$	$V_{se}^S = I_{S,inj}Z_{se2}$
$V_{Re(se)}^M = (R_{se1}I_{M,inj}^Re - X_{se1}I_{M,inj}^Im)$	$V_{Re(se)}^S = (R_{se2}I_{S,inj}^Re - X_{se2}I_{S,inj}^Im)$
$V_{Im(se)}^M = (R_{se1}I_{M,inj}^Im + X_{se1}I_{M,inj}^Re)$	$V_{Im(se)}^S = (R_{se2}I_{S,inj}^Im + X_{se2}I_{S,inj}^Re)$
$ V_{se}^M  = \sqrt{V_{Re(se)}^M + V_{Im(se)}^M}$	$ V_{se}^S  = \sqrt{V_{Re(se)}^S + V_{Im(se)}^S}$
$\Phi_{se}^M = \tan^{-1} \left( \frac{V_{Im(se)}^M}{V_{Re(se)}^M} \right)$	$\Phi_{se}^S = \tan^{-1} \left( \frac{V_{Im(se)}^S}{V_{Re(se)}^S} \right)$

By using the developed current injection IPFC model, only the current mismatches at the ends of IPFC should be changed in load flow algorithm as presented in Eq. (19).

$$\begin{bmatrix}
 \Delta I_{m1} \\
 \Delta I_{r1} \\
 \vdots \\
 \Delta I_{mi} - I_{M,inj}^Im - I_{S,inj}^Im \\
 \Delta I_{ri} - I_{M,inj}^Re - I_{S,inj}^Re \\
 \Delta I_{mj} + I_{M,inj}^Im \\
 \Delta I_{rj} + I_{M,inj}^Re \\
 \Delta I_{mk} + I_{S,inj}^Im \\
 \Delta I_{rk} + I_{S,inj}^Re \\
 \vdots \\
 \Delta I_{mf} \\
 \Delta I_{if} \\
 \Delta V_f^2 \\
 \vdots \\
 \Delta I_{mn} \\
 \Delta I_m
 \end{bmatrix}
 =
 \begin{bmatrix}
 B'_{11} & G'_{11} & \cdots & B_{1i} & G_{1i} & B_{1j} & G_{1j} & B_{1k} & G_{1k} & \cdots & B_{1f} & G_{1f} & 0 & \cdots & B_{1n} & G_{1n} \\
 G''_{11} & B''_{11} & \cdots & G_{1i} & -B_{1i} & G_{1j} & -B_{1j} & G_{1k} & -B_{1k} & \cdots & G_{1f} & -B_{1f} & 0 & \cdots & G_{1n} & -B_{1n} \\
 \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 B_{i1} & G_{i1} & \cdots & B'_{ii} & G'_{ii} & B_{ij} & G_{ij} & B_{ik} & G_{ik} & \cdots & B_{if} & G_{if} & 0 & \cdots & B_{in} & G_{in} \\
 G_{i1} & -B_{i1} & \cdots & G''_{ii} & B''_{ii} & G_{ij} & -B_{ij} & G_{ik} & -B_{ik} & \cdots & G_{if} & -B_{if} & 0 & \cdots & G_{in} & -B_{in} \\
 B_{j1} & G_{j1} & \cdots & B_{ji} & G_{ji} & B'_{jj} & G'_{jj} & B_{jk} & G_{jk} & \cdots & B_{jf} & G_{jf} & 0 & \cdots & B_{jn} & G_{jn} \\
 G_{j1} & -B_{j1} & \cdots & G_{ji} & -B_{ji} & G''_{jj} & B''_{jj} & G_{jk} & -B_{jk} & \cdots & G_{jf} & -B_{jf} & 0 & \cdots & G_{jn} & -B_{jn} \\
 B_{k1} & G_{k1} & \cdots & B_{ki} & G_{ki} & B_{kj} & G_{kj} & B'_{kk} & G'_{kk} & \cdots & B_{kf} & G_{kf} & 0 & \cdots & B_{kn} & G_{kn} \\
 G_{k1} & -B_{k1} & \cdots & G_{ki} & -B_{ki} & G_{kj} & -B_{kj} & G''_{kk} & B''_{kk} & \cdots & G_{kf} & -B_{kf} & 0 & \cdots & G_{kn} & -B_{kn} \\
 \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 B_{f1} & G_{f1} & \cdots & B_{fi} & G_{fi} & B_{ff} & G_{ff} & B_{fk} & G_{fk} & \cdots & B'_{ff} & G'_{ff} & \frac{V_{rf}}{V_f^2} & \cdots & B_{fn} & G_{fn} \\
 G_{f1} & -B_{f1} & \cdots & G_{fi} & -B_{fi} & G_{ff} & -B_{ff} & G_{fk} & -B_{fk} & \cdots & G''_{ff} & B''_{ff} & \frac{-V_{mf}}{V_f^2} & \cdots & G_{fn} & -B_{fn} \\
 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 2V_{rf} & 2V_{mf} & 0 & \cdots & 0 & 0 \\
 \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 B_{n1} & G_{n1} & \cdots & B_{ni} & G_{ni} & B_{nj} & G_{nj} & B_{nk} & G_{nk} & \cdots & B_{nf} & G_{nf} & 0 & \cdots & B'_{nn} & G'_{nn} \\
 G_{n1} & -B_{n1} & \cdots & G_{ni} & -B_{ni} & G_{nj} & -B_{nj} & G_{nk} & -B_{nk} & \cdots & G_{nf} & -B_{nf} & 0 & \cdots & G''_{nn} & B''_{nn}
 \end{bmatrix}
 \begin{bmatrix}
 \Delta V_{r1} \\
 \Delta V_{m1} \\
 \vdots \\
 \Delta V_{ri} \\
 \Delta V_{mi} \\
 \Delta V_{tj} \\
 \Delta V_{mj} \\
 \Delta V_{rk} \\
 \Delta V_{mk} \\
 \vdots \\
 \Delta V_{if} \\
 \Delta V_{mf} \\
 \Delta Q_f \\
 \vdots \\
 \Delta V_{mn} \\
 \Delta V_{mn}
 \end{bmatrix}
 \quad (19)$$

The series voltage parameters of IPFC can be calculated easily during the iterative process. The required equations to calculate these parameters are presented in Table 1.

#### 4. Incorporating of IPFC model in revised NR current injection load flow program

The IPFC current injection model can easily be incorporated in Revised Newton Raphson current injection load flow program. If an IPFC is located in the lines i-l (h) in a power system, the admittance matrix should be modified by adding the reactance equivalent ( $X_{ser1}$  and  $X_{ser2}$ ) of IPFC. The current mismatches should be calculated including the new values of shunt injected currents at the ends of IPFC (buses  $i, j$  and  $k$ ) as shown in Eq. (19). The Jacobian matrix is calculated without change the basic form. The series voltage parameters of IPFC can be calculated during the iterative process as presented in Table 1.

#### 5. Results

Based on the 5-bus, IEEE 57-bus and IEEE 118-bus test systems [12,31], numerical results are carried out to show the effectiveness and performance of the IPFC model. For all the cases, the convergence tolerance is  $10^{-5}$ . The system base MVA is 100. All the test programmes are written in C++. The SuperLU library is used to perform all matrix calculations associated with the power-flow solution process [32].

##### 5.1. 5-Bus test system

The objective of this test is to validate the developed IPFC model into Revised Newton Raphson current injection load flow method. The same data of 5-bus test system in [12] is used here. The test network is incorporated with IPFC between lines 4–3(5) as shown in Fig. 5. The purpose of IPFC (the reactance of series transformers is set at 0.1 p.u) on this network is to maintain active and reactive powers leaving IPFC towards buses 3(5) to be 30 MW, 2 MVAR and 25 MW, respectively. The results of voltages and phase angles of the network system and IPFC sources are presented in Table 2. Whereas, the result of power flow on the network is presented in Fig. 5.

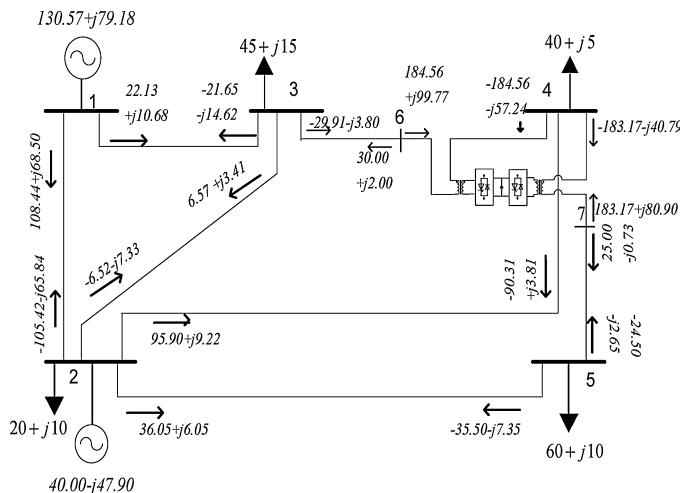


Fig. 5. Load flow solution of 5 bus test system with IPFC model.

**Table 2**

The voltages of 5-bus test system with IPFC model.

Bus number	$ V $ (p.u)	$\varphi$ (deg.)
1	1.060	0.000
2	1.000	-2.740
3	1.014	-2.258
4	0.937	-12.936
5	0.977	-5.100
Auxiliary bus (6)	1.017	-1.774
Auxiliary bus (7)	1.000	-1.663
Master series source	0.233	62.892
Slave series source	0.223	67.318

The power flows result (as shown in Fig. 5) has justified the capability of Revised Newton Raphson current injection method in solving load flow considering IPFC model in the network. All the obtained values are fulfilling the specific control requirements of power flow that can be proven by calculating the power flow between bus 6 and 4 by using the obtained voltage and phase angle of related buses.

## 5.2. IEEE 57-bus test system

Based on the IEEE 57-bus system, many cases are used to exhibit the power flow control capability of the IPFC model into Revised Newton Raphson current injection load flow method. The IPFC on the lines 15–13(14) is used to maintain the active and reactive powers leaving IPFC towards buses 13(14) to be 35 MW, 5 MVAR and 25 MW, respectively, while the reactance of series transformers is set at different values. From Table 3, it can be observed that the IPFC model can be operated at different size of series transformer reactance. Where, there is inverse relationship between the shunt injected current and the series transformer reactance.

The series voltage parameters of IPFC model and the shunt injected currents with various values of specified line power flow are given in Table 4. It can be observed that the injected currents parameters of IPFC model are affected by the specified line power flow. Also the result shows the ability of the proposed IPFC model to operate at different specified power flow.

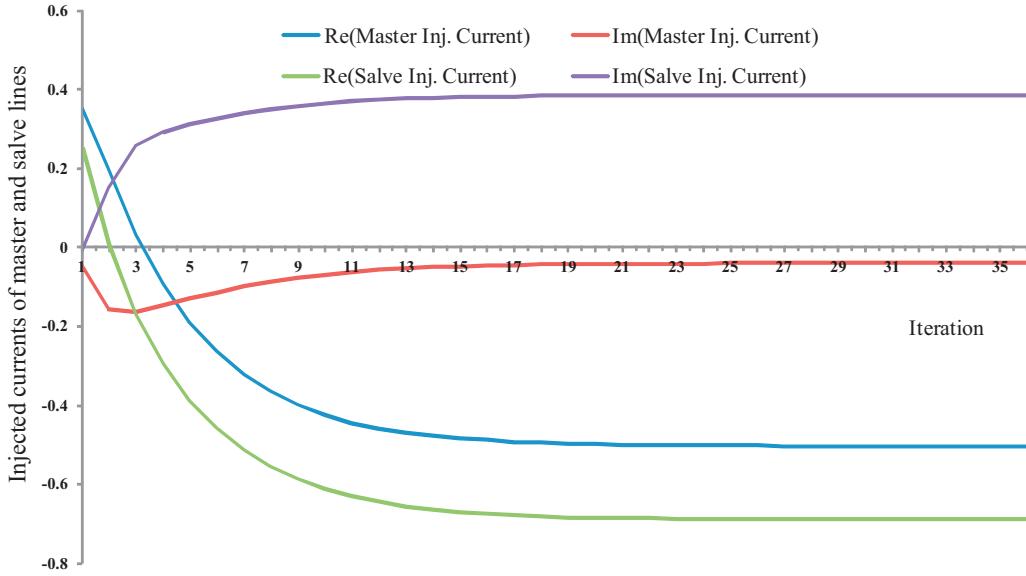
Fig. 6 shows the variation of real and imaginary components of shunt injected currents of master and slave lines against the iterations. It can be observed that the injected currents change rapidly for the first few iterations and reach to the final value after a few more iterations compared to the uncontrolled case.

Fig. 7 shows the variation of the real and imaginary components of series voltage parameters of IPFC model against the iterations. It can be observed that the series voltage parameters change smoothly to the final values without oscillation. The convergence happened after a few more iteration compared to the uncontrolled case.

**Table 3**

Performance of IEEE 57-bus system with IPFC model at different values of series reactance.

Xe (p.u)	Injection currents at IPFCs terminals				IPFC parameters			
	$I_{M,inj}^{Re}$ (p.u)	$I_{M,inj}^{Im}$ (p.u)	$I_{S,inj}^{Re}$ (p.u)	$I_{S,inj}^{Im}$ (p.u)	Series source (1)		Series source (2)	
					$ V_{se}^M $ (p.u)	$\Phi_{se}^M$ (deg.)	$ V_{se}^S $ (p.u)	$\Phi_{se}^S$ (deg.)
0.20	-0.084	-0.078	-0.212	0.193	0.023	-46.862	0.057	47.726
0.15	-0.223	-0.066	-0.371	0.258	0.035	-73.687	0.068	55.284
0.10	-0.503	-0.040	-0.689	0.386	0.050	-85.508	0.079	60.765
0.05	-1.343	0.037	-1.643	0.773	0.067	88.465	0.091	64.855
0.01	-8.057	0.653	-9.275	3.862	0.081	85.413	0.100	67.427



**Fig. 6.** Convergence characteristics of injected currents at the ends of IPFC model.

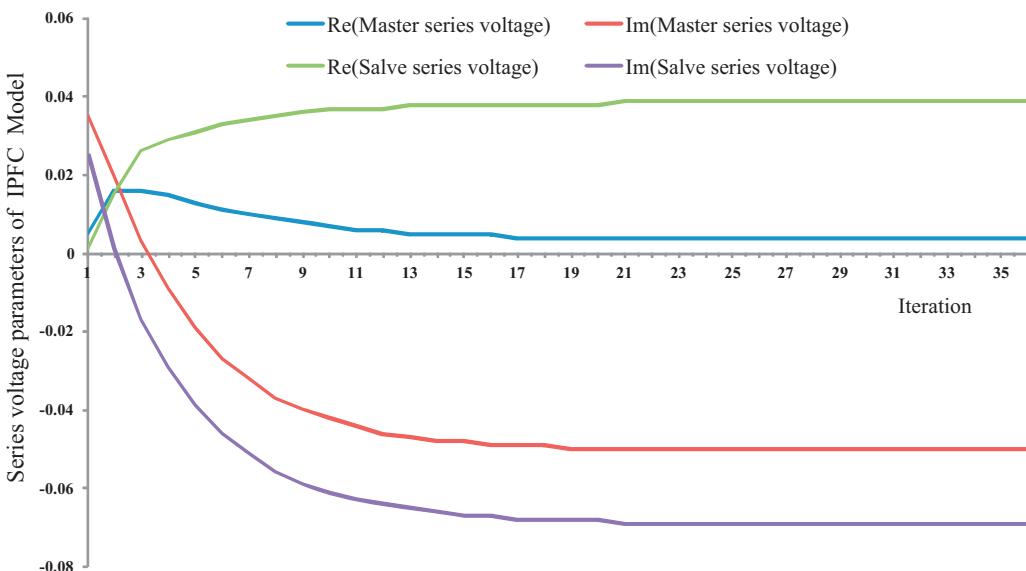
### 5.3. IEEE 118 bus test system

The IEEE 118 bus is selected as a large scale system. The IPFC is placed on the lines 94–93(95) to control the active and reactive powers leaving IPFC towards buses 93 and 95 at different values as presented in [Table 5](#). Based on this test, the ability of IPFC model to operate at different specified power flow is validated.

**Table 4**

IEEE 57-bus system with IPFC model at different values of specified power flow.

Specified power			Injected currents at ends of IPFCs				IPFC parameters			
$P_{sp}^M$ (MW)	$Q_{sp}^M$ (MVAR)	$P_{sp}^S$ (MW)	$I_{M,inj}^{Re}$ (p.u)	$I_{M,inj}^{Im}$ (p.u)	$I_{S,inj}^{Re}$ (p.u)	$I_{S,inj}^{Im}$ (p.u)	Series source (1)		Series source (2)	
							$ V_{se}^M $ (p.u)	$\Phi_{se}^M$ (deg.)	$ V_{se}^S $ (p.u)	$\Phi_{se}^S$ (deg.)
50	6	60	0.361	-0.254	0.542	0.129	0.044	54.840	0.056	-76.638
-60	-4	-34	-3.878	1.791	-3.643	1.353	0.427	65.243	0.389	69.660
-40	8.4	-90	-4.211	1.429	-5.468	1.605	0.445	71.298	0.570	73.683
70	-10.5	-110	-1.043	0.319	-4.801	1.206	0.109	73.030	0.495	75.942
0	0	25	-1.499	0.449	-1.056	0.560	0.156	73.357	0.120	62.080



**Fig. 7.** Convergence characteristics of series voltage parameters of IPFC model.

**Table 5**

IEEE 118-bus system with IPFC model at different values of specified power flow.

Specified power			Injected currents at ends of IPFCs				IPFC parameters			
$P_{sp}^M$ (MW)	$Q_{sp}^M$ (MVAR)	$P_{sp}^S$ (MW)	$I_{M,inj}^{Re}$ (p.u)	$I_{M,inj}^{Im}$ (p.u)	$I_{S,inj}^{Re}$ (p.u)	$I_{S,inj}^{Im}$ (p.u)	Series source (1)		Series source (2)	
							$ V_{se}^M $ (p.u)	$\Phi_{se}^M$ (deg.)	$ V_{se}^S $ (p.u)	$\Phi_{se}^S$ (deg.)
130	66	170	5.545	1.758	2.961	-1.206	0.582	-72.445	0.320	67.874
150	72	-57	5.496	2.514	-3.249	-0.136	0.604	-65.451	0.325	-87.643
82	-30	45	2.728	3.235	-0.470	-0.558	0.423	-40.159	0.073	-40.114
-100	37	29	-1.870	-1.403	-1.180	-0.711	0.234	-53.149	0.138	-58.953
-77	-52	-39	-2.537	1.367	-3.181	-0.199	0.288	61.723	0.319	-86.469

**Table 6**

IEEE 118-bus system with single and multiple IPFC controllers.

Case	IPFC lines	$I_{M,inj}$ (p.u)	$I_{S,inj}$ (p.u)	Auxiliary voltage bus		IPFC parameters	
				Bus (j)	Bus (k)	$V_{se}^M$ (p.u)	$V_{se}^S$ (p.u)
A	96/82(94)	$1.252 + j0.189$	$2.648 + j0.643$	$0.987 \angle 24.961^\circ$	$1.007 \angle 34.886^\circ$	$0.127 \angle -81.481^\circ$	$0.273 \angle -76.399^\circ$
B	96/82(94)	$0.468 - j0.756$	$2.931 + j0.831$	$1.027 \angle 20.949^\circ$	$1.005 \angle 38.883^\circ$	$0.089 \angle 31.774^\circ$	$0.307 \angle -75.372^\circ$
	83/82(84)	$-0.896 - j2.584$	$3.028 - j0.573$	$1.023 \angle 20.436^\circ$	$1.064 \angle 45.960^\circ$	$0.274 \angle -19.134^\circ$	$0.081 \angle 44.812^\circ$
C	96/82(94)	$2.489 - j0.019$	$8.342 + j4.428$	$1.027 \angle 20.127^\circ$	$1.061 \angle 61.750^\circ$	$0.249 \angle 89.617^\circ$	$0.199 \angle -59.604^\circ$
	83/82(84)	$-2.915 - j6.843$	$4.970 - j0.962$	$1.023 \angle 19.614^\circ$	$1.161 \angle 72.004^\circ$	$0.347 \angle -23.089^\circ$	$0.375 \angle 81.186^\circ$
	94/93(95)	$1.253 + j3.374$	$-6.809 - j4.485$	$1.013 \angle 78.309^\circ$	$0.949 \angle 10.257^\circ$	$0.360 \angle -20.385^\circ$	$0.264 \angle -14.655^\circ$

Also the ability of the IPFC model into Revised Newton Raphson current injection load flow to solve multiple IPFC in the system is validated. In this test, the placing of IPFCs on this system is chosen randomly. The position of IPFC is as shown in Fig. 1. The IPFC is placed on the lines i–l (h) to maintain active and reactive powers leaving IPFC towards buses l and h to be 60 MW, 3 MVAR and 30 MW, respectively, while the series transformer reactance is set to be 0.1 p.u. The results of this test are presented in Table 6.

## 6. Conclusions

This paper has presented an easy modelling of IPFC into Revised Newton Raphson current injection load flow analysis. The model is based on shunt injected currents at the ends of IPFC. These injected currents are updated during the iterative process according to the desired power flow, correction of buses voltage and system reactive power flow between the second auxiliary and received buses. By using this model, the original structure and symmetry of admittance matrix can still be kept. The incorporating of IPFC in load flow becomes easy without changing in the basic computational algorithm. Consequently, the complexities of the computer program codes are reduced. The IPFC parameters can be checked and if their values are within the limits or not during the iterative process. Also the developed model overcomes the problem which exists when only the IPFC links between two sub-networks. The Revised Newton current injection power flow program with this IPFC model is useful tool for power system planning and operation control of large scale power system. The developed IPFC model has been validated on the literature 5-bus, IEEE 57-bus and 118-bus systems with excellent performance characteristics

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